

## GREEN'S RELATIONS AND THEIR GENERALIZATIONS ON SEMIGROUPS

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Dedicated to the 65th birthday of Klaus Denecke

### Abstract

Green's relations and their generalizations on semigroups are useful in studying regular semigroups and their generalizations. In this paper, we first give a brief survey of this topic. We then give some examples to illustrate some special properties of generalized Green's relations which are related to completely regular semigroups and abundant semigroups.

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