Discussiones Mathematicae
General Algebra and Applications 30 (2010) 7–33
doi:10.7151/dmgaa.1160

THE SUBMAXIMAL CLONES ON THE THREE-ELEMENT SET WITH FINITELY MANY RELATIVE $\mathcal{R}$-CLASSES*

Erkko Lehtonen

University of Luxembourg
6, rue Richard Coudenhove-Kalergi
L–1359 Luxembourg, Luxembourg

e-mail: erkko.lehtonen@uni.lu

and

Ágnes Szendrei

Department of Mathematics
University of Colorado at Boulder
Campus Box 395
Boulder, CO 80309–0395, USA
and
Bolyai Institute
Aradi vértanúk tere 1, H–6720 Szeged, Hungary

e-mail: szendrei@euclid.colorado.edu

Abstract

For each clone $\mathcal{C}$ on a set A there is an associated equivalence relation analogous to Green's $\mathcal{R}$-relation, which relates two operations on A if and only if each one is a substitution instance of the other using operations from $\mathcal{C}$. We study the maximal and submaximal clones on a three-element set and determine which of them have only finitely many relative $\mathcal{R}$-classes.

Keywords: clone, maximal clone, submaximal clone, Green's relations.

2000 Mathematics Subject Classification: 08A55, 03B10.

*This material is based upon work supported by the Hungarian National Foundation for Scientific Research (OTKA) grants no. T 048809 and K60148.

REFERENCES

[1] G.A. Burle, *The classes of k-valued logics containing all one-variable functions*, Diskretnyi Analiz **10** (1967), 3–7.

[2] J. Demetrovics and J. Bagyinszki, The lattice of linear classes in prime-valued logics, in: J.L. Kulikowski, M. Michalewicz, S.V. Yablonskii, Yu.I. Zhuravlev (eds.), Discrete Mathematics (Warsaw, 1977), Banach Center Publ. 7, PWN, Warsaw (1982), pp. 105–123.

[3] A. Feigelson and L. Hellerstein, *The forbidden projections of unate functions*, Discrete Appl. Math. **77** (1997), 221–236. doi:10.1016/S0166-218X(96)00136-9

[4] M.A. Harrison, *On the classification of Boolean functions by the general linear and affine groups*, J. Soc. Indust. Appl. Math. **12** (2) (1964), 285–299. doi:10.1137/0112026

[5] J. Henno, *Green's equivalences in Menger systems,* Tartu Riikl. Ül. Toimetised **277** (1971), 37–46.

[6] D. Lau, *Submaximale Klassen von P_3*, Elektron. Informationsverarb. Kybernet. **18** (1982), 227–243.

[7] D. Lau, *Function Algebras on Finite Sets,* Springer-Verlag, Berlin, Heidelberg 2006.

[8] E. Lehtonen, *Descending chains and antichains of the unary, linear, and monotone subfunction relations*, Order **23** (2006), 129–142. doi:10.1007/s11083-006-9036-y

[9] E. Lehtonen and Á. Szendrei, *Equivalence of operations with respect to discriminator clones*, Discrete Math. **309** (2009), 673–685. doi:10.1016/j.disc.2008.01.003

[10] E. Lehtonen and Á. Szendrei, *Clones with finitely many relative $\mathcal{R}$-classes*, arXiv:0905.1611.

[11] H. Machida, On closed sets of three-value monotone logical functions, in: B. Csákány, I. Rosenberg (eds.), Finite Algebra and Multiple-Valued Logic (Szeged, 1979), Colloq. Math. Soc. János Bolyai 28, North-Holland, Amsterdam (1981), pp. 441–467.

[12] S.S. Marchenkov, J. Demetrovics and L. Hannák, *Closed classes of self-dual functions in P_3,* Metody Diskret. Anal. **34** (1980), 38–73.

[13] N. Pippenger, *Galois theory for minors of finite functions*, Discrete Math. **254** (2002), 405–419. doi:10.1016/S0012-365X(01)00297-7

[14] R. Pöschel and L.A. Kalužnin, *Funktionen- und Relationenalgebren: Ein Kapitel der diskreten Mathematik,* Birkhäuser, Basel, Stuttgart 1979.

[15] I.G. Rosenberg, *Über die funktionale Vollständigkeit in den mehrwertigen Logiken*, Rozpravy Československé Akad. Věd, Řada Mat. Přírod. Věd **80** (1970), 3–93.

[16] J. Słupecki, *Kryterium pełności wielowartościowych systemów logiki zdań*, C.R. Séanc. Soc. Sci. Varsovie, Cl. III **32** (1939), 102–109. English translation: *A criterion of fullness of many-valued systems of propositional logic*, Studia Logica **30** (1972), 153–157. doi:10.1007/BF02120845

[17] Á. Szendrei, *Clones in Universal Algebra,* Séminaire de mathématiques supérieures **99**, Les Presses de l'Université de Montréal, Montréal 1986.

[18] C. Wang, *Boolean minors*, Discrete Math. **141** (1995), 237–258. doi:10.1016/0012-365X(93)E0191-6

[19] C. Wang and A.C. Williams, *The threshold order of a Boolean function*, Discrete Appl. Math. **31** (1991), 51–69. doi:10.1016/0166-218X(91)90032-R

[20] S.V. Yablonsky, *Functional constructions in a k-valued logic*, Trudy Mat. Inst. Steklov. **51** (1958), 5–142.

[21] I.E. Zverovich, *Characterizations of closed classes of Boolean functions in terms of forbidden subfunctions and Post classes*, Discrete Appl. Math. **149** (2005), 200–218. doi:10.1016/j.dam.2004.06.028

Received 1 May 2009
Revised 30 July 2009