

THE SUBMAXIMAL CLONES ON THE
THREE-ELEMENT SET WITH FINITELY
MANY RELATIVE \mathcal{R} -CLASSES*

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Abstract

For each clone \mathcal{C} on a set A there is an associated equivalence relation analogous to Green's \mathcal{R} -relation, which relates two operations on A if and only if each one is a substitution instance of the other using operations from \mathcal{C} . We study the maximal and submaximal clones on a three-element set and determine which of them have only finitely many relative \mathcal{R} -classes.

Keywords: clone, maximal clone, submaximal clone, Green's relations.

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