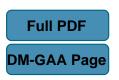
Discussiones Mathematicae General Algebra and Applications 29 (2009) 81–107 doi:10.7151/dmgaa.1152



SPECIAL M-HYPERIDENTITIES IN BIREGULAR LEFTMOST GRAPH VARIETIES OF TYPE (2,0)

APINANT ANANTPINITWATNA AND TIANG POOMSA-ARD

Department of Mathematics, Faculty of Science Mahasarakham University, Mahasarakham 44150, Thailand

e-mail: tiang@kku.ac.th

Abstract

Graph algebras establish a connection between directed graphs without multiple edges and special universal algebras of type (2.0). We say that a graph G satisfies a term equation $s \approx t$ if the corresponding graph algebra A(G) satisfies $s \approx t$. A class of graph algebras V is called a graph variety if $V = Mod_g\Sigma$ where Σ is a subset of $T(X) \times T(X)$. A graph variety $V' = Mod_q\Sigma'$ is called a biregular leftmost graph variety if Σ' is a set of biregular leftmost term equations. A term equation $s \approx t$ is called an identity in a variety V if A(G) satisfies $s \approx t$ for all $G \in V$. An identity $s \approx t$ of a variety V is called a hyperidentity of a graph algebra A(G), $G \in V$ whenever the operation symbols occurring in s and t are replaced by any term operations of A(G) of the appropriate arity, the resulting identities hold in A(G). An identity $s \approx t$ of a variety V is called an M-hyperidentity of a graph algebra A(G), $G \in V$ whenever the operation symbols occurring in s and t are replaced by any term operations in a subgroupoid M of term operations of A(G)of the appropriate arity, the resulting identities hold in A(G).

In this paper we characterize special M-hyperidentities in each biregular leftmost graph variety. For identities, varieties and other basic concepts of universal algebra see e.g. [3].

Keywords: varieties, biregular leftmost graph varieties, identities, term, hyperidentity, M-hyperidentity, binary algebra, graph algebra.

2000 Mathematics Subject Classification: 05C25, 08B15.

References

- [1] A. Ananpinitwatna and T. Poomsa-ard, *Identities in biregular leftmost graph varieties of type* (2,0), Asian-European J. of Math. **2** (1) (2009), 1–18.
- [2] A. Ananpinitwatna and T. Poomsa-ard, Hyperidentities in biregular leftmost graph varieties of type (2,0), Int. Math. Forum 4 (18) (2009), 845–864.
- [3] K. Denecke and S.L. Wismath, *Universal Algebra and Applications in Theo*retical Computer Science, Chapman and Hall/CRC 2002.
- [4] K. Denecke and T. Poomsa-ard, *Hyperidentities in graph algebras*, Contributions to General Algebra and Aplications in Discrete Mathematics, Potsdam (1997), 59–68.
- [5] K. Denecke, D. Lau, R. Pöschel and D. Schweigert, *Hyperidentities, hyperequational classes and clone congruences*, Verlag Hölder-Pichler-Tempsky, Wien, Contributions to General Algebra 7 (1991), 97–118.
- [6] M. Krapeedang and T. Poomsa-ard, Biregular leftmost graph varieties of type (2,0), accepted to publish in AAMS.
- [7] E.W. Kiss, R. Pöschel and P. Pröhle, Subvarieties of varieties generated by graph algebras, Acta Sci. Math. **54** (1990), 57–75.
- [8] J.Khampakdee and T. Poomsa-ard, Hyperidentities in $(xy)x \approx x(yy)$ graph algebras of type (2,0), Bull. Khorean Math. Soc. 44 (4) (2007), 651–661.
- [9] J. Płonka, *Hyperidentities in some of vareties*, pp. 195–213 in: General Algebra and discrete Mathematics ed. by K. Denecke and O. Lüders, Lemgo 1995.
- [10] J. Płonka, Proper and inner hypersubstitutions of varieties, pp. 106–115 in: ,,Proceedings of the International Conference: Summer School on General Algebra and Ordered Sets 1994", Palacký University Olomouce 1994.
- [11] T. Poomsa-ard, *Hyperidentities in associative graph algebras*, Discussiones Mathematicae General Algebra and Applications **20** (2) (2000), 169–182.
- [12] T. Poomsa-ard, J. Wetweerapong and C. Samartkoon, Hyperidentities in idempotent graph algebras, Thai Journal of Mathematics 2 (2004), 171–181.
- [13] T. Poomsa-ard, J. Wetweerapong and C. Samartkoon, *Hyperidentities in transitive graph algebras*, Discussiones Mathematicae General Algebra and Applications **25** (1) (2005), 23–37.
- [14] R. Pöschel, *The equational logic for graph algebras*, Zeitschr.f.math. Logik und Grundlagen d. Math. Bd. S. **35** (1989), 273–282.
- [15] R. Pöschel, Graph algebras and graph varieties, Algebra Universalis 27 (1990), 559–577.

- [16] R. Pöschel and W. Wessel, Classes of graph definable by graph algebras identities or quasiidentities, Comment. Math. Univ, Carolinae 28 (1987), 581–592.
- [17] C.R. Shallon, Nonfinitely based finite algebras derived from lattices, Ph. D. Dissertation, Uni. of California, Los Angeles 1979.

Received 30 April 2009 Revised 09 August 2009