

SPECIAL M-HYPERIDENTITIES IN BIREGULAR LEFTMOST GRAPH VARIETIES OF TYPE (2,0)

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Abstract

Graph algebras establish a connection between directed graphs without multiple edges and special universal algebras of type (2,0). We say that a graph G satisfies a term equation $s \approx t$ if the corresponding graph algebra $A(G)$ satisfies $s \approx t$. A class of graph algebras V is called a graph variety $\overline{V} = \text{Mod}_g \Sigma$ where Σ is a subset of $T(X) \times T(X)$. A graph variety $V' = \text{Mod}_g \Sigma'$ is called a biregular leftmost graph variety if Σ' is a set of biregular leftmost term equations. A term equation $s \approx t$ is called an identity in a variety V if $A(G)$ satisfies $s \approx t$ for all $G \in V$. An identity $s \approx t$ of a variety V is called a hyperidentity of a graph algebra $A(G)$, $G \in V$ whenever the operation symbols occurring in s and t are replaced by any term operations of $A(G)$ of the appropriate arity, the resulting identities hold in $A(G)$. An identity $s \approx t$ of a variety V is called an M -hyperidentity of a graph algebra $A(G)$, $G \in V$ whenever the operation symbols occurring in s and t are replaced by any term operations in a subgroupoid M of term operations of $A(G)$ of the appropriate arity, the resulting identities hold in $A(G)$.

In this paper we characterize special M -hyperidentities in each biregular leftmost graph variety. For identities, varieties and other basic concepts of universal algebra see e.g. [3].

Keywords: varieties, biregular leftmost graph varieties, identities, term, hyperidentity, M -hyperidentity, binary algebra, graph algebra.

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