

## ON THE MATRIX NEGATIVE PELL EQUATION

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### Abstract

Let  $N$  be a set of natural numbers and  $Z$  be a set of integers. Let  $M_2(Z)$  denotes the set of all  $2 \times 2$  matrices with integer entries.

We give necessary and sufficient conditions for solvability of the matrix negative Pell equation

$$(P) \quad X^2 - dY^2 = -I \quad \text{with } d \in N$$

for nonsingular  $X, Y$  belonging to  $M_2(Z)$  and his generalization

$$(P_n) \quad \sum_{i=1}^n X_i^2 - d \sum_{i=1}^n Y_i^2 = -I \quad \text{with } d \in N$$

for nonsingular  $X_i, Y_i \in M_2(Z), i = 1, \dots, n$ .

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