

## ON COVARIETY LATTICES

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### Abstract

This paper shows basic properties of covariety lattices. Such lattices are shown to be infinitely distributive. The covariety lattice  $L_{CV}(\mathbf{K})$  of subcovarieties of a covariety  $\mathbf{K}$  of  $F$ -coalgebras, where  $F : \mathbf{Set} \rightarrow \mathbf{Set}$  preserves arbitrary intersections is isomorphic to the lattice of subcoalgebras of a  $\mathcal{P}_\kappa$ -coalgebra for some cardinal  $\kappa$ . A full description of the covariety lattice of  $\mathcal{I}d$ -coalgebras is given. For any topology  $\tau$  there exist a bounded functor  $F : \mathbf{Set} \rightarrow \mathbf{Set}$  and a covariety  $\mathbf{K}$  of  $F$ -coalgebras, such that  $L_{CV}(\mathbf{K})$  is isomorphic to the lattice  $(\tau, \cup, \cap)$  of open sets of  $\tau$ .

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