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ON SOME PROPERTIES OF CHEBYSHEV POLYNOMIALS

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Abstract

Letting T_n (resp. U_n) be the n-th Chebyshev polynomials of the first (resp. second) kind, we prove that the sequences $\left(X^k T_{n-k}\right)_k$ and $\left(X^k U_{n-k}\right)_k$ for $n - 2\lfloor n/2 \rfloor \leq k \leq n - \lfloor n/2 \rfloor$ are two basis of the $\mathbb{Q}$-vectorial space $\mathbb{E}_n[X]$ formed by the polynomials of $\mathbb{Q}[X]$ having the same parity as n and of degree $\leq n$. Also T_n and U_n admit remarkableness integer coordinates on each of the two basis.

Keywords: Chebyshev polynomials, integer coordinates.

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