

INTERIOR AND CLOSURE OPERATORS ON BOUNDED COMMUTATIVE RESIDUATED ℓ -MONOIDS

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Abstract

Topological Boolean algebras are generalizations of topological spaces defined by means of topological closure and interior operators, respectively. The authors in [14] generalized topological Boolean algebras to closure and interior operators of MV -algebras which are an algebraic counterpart of the Łukasiewicz infinite valued logic. In the paper, these kinds of operators are extended (and investigated) to the wide class of bounded commutative $R\ell$ -monoids that contains e.g. the classes of BL -algebras (i.e., algebras of the Hájek's basic fuzzy logic) and Heyting algebras as proper subclasses.

Keywords: residuated ℓ -monoid, residuated lattice, closure operator, BL -algebra, MV -algebra.

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