

## REMARKS TO GŁAZEK'S RESULTS ON $n$ -ARY GROUPS

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### Abstract

This is a survey of the results obtained by K. Głazek and his co-workers. We restrict our attention to the problems of axiomatizations of  $n$ -ary groups, classes of  $n$ -ary groups, properties of skew elements and homomorphisms induced by skew elements, constructions of covering groups, classifications and representations of  $n$ -ary groups. Some new results are added too.

**Key words and phrases:** polyadic group,  $n$ -ary group, variety, representation.

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