

## PSEUDOCOMPLEMENTS IN SUM-ORDERED PARTIAL SEMIRINGS

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### Abstract

We study a particular way of introducing pseudocomplementation in ordered semigroups with zero, and characterise the class of those pseudocomplemented semigroups, termed *g*-semigroups here, that admit a Glivenko type theorem (the pseudocomplements form a Boolean algebra). Some further results are obtained for *g*-semirings – those sum-ordered partially additive semirings whose multiplicative part is a *g*-semigroup. In particular, we introduce the notion of a partial Stone semiring and show that several well-known elementary characteristics of Stone algebras have analogues for such semirings.

**Keywords:** Glivenko theorem, partial monoid, partial semiring, pseudocomplementation, semigroup, Stone semiring, sum-ordering.

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