

AXIOMATIZATION OF QUASIGROUPS

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Abstract

Quasigroups were originally described combinatorially, in terms of existence and uniqueness conditions on the solutions to certain equations. Evans introduced a universal-algebraic characterization, as algebras with three binary operations satisfying four identities. Now, quasigroups are redefined as heterogeneous algebras, satisfying just two conditions respectively known as hypercommutativity and hypercancellativity.

Keywords: quasigroup, heterogeneous algebra, hyperidentity.

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