

ON THE MAXIMAL SUBSEMIGROUPS  
OF THE SEMIGROUP OF ALL  
MONOTONE TRANSFORMATIONS

ILIYA GYUDZHENOV AND ILINKA DIMITROVA<sup>1</sup>

*South-West University "Neofit Rilski"*  
*Faculty of Mathematics and Natural Science*  
*2700 Blagoevgrad, Bulgaria*

**e-mail:** iliadg@aix.swu.bg

**e-mail:** ilinka\_dimitrova@yahoo.com

**Abstract**

In this paper we consider the semigroup  $M_n$  of all monotone transformations on the chain  $X_n$  under the operation of composition of transformations. First we give a presentation of the semigroup  $M_n$  and some propositions connected with its structure. Also, we give a description and some properties of the class  $\tilde{J}_{n-1}$  of all monotone transformations with rank  $n - 1$ . After that we characterize the maximal subsemigroups of the semigroup  $M_n$  and the subsemigroups of  $M_n$  which are maximal in  $\tilde{J}_{n-1}$ .

**Keywords:** transformation semigroup, maximal subsemigroups, idempotent, isotone, antitone and monotone transformations, Green's equivalences.

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