

ALMOST ASSOCIATIVE OPERATIONS GENERATING A MINIMAL CLONE

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Abstract

Characterizations of ‘almost associative’ binary operations generating a minimal clone are given for two interpretations of the term ‘almost associative’. One of them uses the associative spectrum, the other one uses the index of nonassociativity to measure how far an operation is from being associative.

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